Effects of shallow slope on the evolution of numerically simulated nocturnal low-level jets

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Nocturnal low-level jets over U.S. Great Plains
(examples from Klein et al. BLM 2016)
Flow variables in slope-following coordinates

\[ u, v, w \] – velocity components along slope-following coordinates \( x, y, z \);

\[ b = g[\Theta - \Theta_e(z')] / \Theta_r \] – buoyancy; \( \Theta_e(z') \) – environmental potential temperature;

\[ \pi = [p - p_e(z')] / \rho_r \] – normalized \( p \); \( \gamma = d\Theta_e / dz' > 0 \) – prescribed \( \Theta_e \) gradient;

\( f > 0 \) – Coriolis parameter, \( V_g \) – geostrophic wind (known and directed along \( y \));

\[ N = (g\gamma / \Theta_r)^{1/2} \] – Brunt-Väisälä frequency; \( \alpha \) – slope angle \( \sim \pi / 1000 = 0.18^\circ \);

\( \nu \) – kinematic viscosity, \( \nu_h \) – thermal diffusivity (we take \( \nu_h = \nu \), so \( \text{Pr} = \nu / \nu_h \) is 1).
Governing equations for boundary-layer flow on a slope

Momentum balance in Boussinesq approximation:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{\partial \pi}{\partial x} + f (v - V_g) - b \sin \alpha + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right),
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = - \frac{\partial \pi}{\partial y} - f u + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right),
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{\partial \pi}{\partial z} + b \cos \alpha + \nu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right),
\]

Thermal energy (buoyancy balance):

\[
\frac{\partial b}{\partial t} + u \frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y} + w \frac{\partial b}{\partial z} = \frac{N^2}{\nu_h} \left( u \sin \alpha - w \cos \alpha \right) + \nu_h \left( \frac{\partial^2 b}{\partial x^2} + \frac{\partial^2 b}{\partial y^2} + \frac{\partial^2 b}{\partial z^2} \right),
\]

Mass conservation (continuity):

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.
\]
Simulation setup

Boundary conditions

**Lateral** \((x-y)\) conditions for \(u, v, w, b,\) and \(\pi\) are periodic.

**Upper** conditions (large \(z\)): \(\partial \varphi / \partial z = 0,\) where \(\varphi = (u, v, b); \ w = 0,\) and \(\partial \pi / \partial z\) from the third equation of motion (3).

**Lower** conditions \((z = 0)\): no-slip/impermeability \((u = v = w = 0),\) \(\partial \pi / \partial z\) from (3), and \(b = b_s(t)\) or \(B = -\nu (\partial b / \partial z)\big|_{z=0} = B_s(t).\)

Initial state

Stably stratified atmosphere in the geostrophic equilibrium with linear \(\Theta_e(z')\) and zero \(b.\)

Surface buoyancy forcing

At \(0 < t \leq \Delta t_d:\) \(b_{sd} = \text{const} > 0\) or \(B_{sd} = \text{const} > 0.\)

At \(t > \Delta t_d:\) \(b_{sn} = \text{const} < b_{sd}\) or \(B_{sn} = \text{const} < 0.\)
Simulation domain and grid

Dimensions ($L_i$, rescaled to VAT) and grid node numbers $N_i$:

$L_x, L_y = 1024-4096$ m; $N_x, N_y = 256-1024$; $L_z = 1536$ m, $N_z = 384$

Numerics: Micro HH ($4^3$), developed by Chiel van Heerwaarden, with 4th order Runge-Kutta time integration, 4th order advection, and 4th order diffusion.
Slope-angle sensitivity of TKE and $V$: $\alpha = 0^\circ, 0.09^\circ, 0.18^\circ, 0.27^\circ$

Surface buoyancy forcing case
256x256x384 (slope) vs. 256x256x384 (flat) - time: 660 m
Slope-angle sensitivity of TKE and $V$: $\alpha = 0^\circ, 0.09^\circ, 0.18^\circ, 0.27^\circ$

Surface buoyancy flux forcing case
$|V_0| = 20 \text{ m/s}$, $256 \times 256 \times 384$ ($\omega = -2\tilde{U}_0(\text{slope})$, $-2\tilde{U}_0(\text{flat})$) - time: 360 m
$V_0 = 20 \text{ m/s, } 256 \times 256 \times 384 (\bar{\overline{W}}=-2V_{y}(\text{slope}), -2V_{y}(\text{flat}))$ - time: 421 m
$|V_g| = 20 \text{ m/s, } 256 \times 256 \times 384 \text{ (over } (-2\mathcal{V}_s, -2\mathcal{V}_f) \text{)} \cdot \text{time: 541 m}$
$|V| = 20 \text{ m/s}, 256 \times 256 \times 384$ ($\vec{v} = -2V(\text{slope}), -2V(\text{flat})$). Time: 660 m
\[ |V_g| = 20 \text{ m/s}, 256 \times 256 \times 384 \] 
\[ \frac{\theta, \text{slope}}{\theta, \text{flat}}, \frac{v, \text{slope}}{v, \text{flat}}, \frac{|V|, \text{slope}}{|V|, \text{flat}} \]

\[ \text{time: 841 m} \]
$|V_0| = 20 \text{ m/s}$, 256x256x384 ($\overline{uv} = -2\overline{V}([\text{slope}], -2\overline{V}([\text{flat}])) \cdot \text{time}: 899 \text{ m}$

- Left panel: $b, \theta$ $(g, \theta_0)$
- Middle panel: $(u, v, |V|)/|V_0|$
- Right panel: $(u^2, e)/|V_0|^2$
No external geostrophic forcing is needed for LLJ on a slope!

Setup: $\alpha = 0.18^\circ$, $V_g = 0$, $b_{sd} = 0.1 \text{ m s}^{-2}$, $b_{sn} = 0$, $N = 2 \times 10^{-2} \text{ s}^{-1}$
Conclusions

- Shallow slope affects the LLJ structure and evolution in multiple ways.
- Along-slope advection of environmental potential temperature can turn a quiet night into a turbulent one, leading to drastic changes in the LLJ properties.
- Over a slope, a pronounced nighttime LLJ can develop in the absence of external geostrophic forcing.
- LLJ evolution over a slope depends on the type of surface forcing (buoyancy versus buoyancy flux) much stronger than evolution of the flat-terrain LLJ.
- Daytime flow preconditioning plays especially important role in LLJ over a slope.

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Observed nocturnal LLJ over Great Plains

October 24, 2012, northern Oklahoma

Wind speed from Doppler lidar scans
Scaled governing equations

**Scales:** \( V = \left| V_g \right| \) velocity; \( H \) (boundary-layer depth \( \sim \) domain height) length; \( HV^{-1} \) time; \( V^2H^{-1} \) buoyancy; \( V^2 \) pressure.

**Normalized governing equations:**

\[
\frac{\partial u_n}{\partial t_n} + u_n \frac{\partial u_n}{\partial x_n} + v_n \frac{\partial u_n}{\partial y_n} + w_n \frac{\partial u_n}{\partial z_n} = -\frac{\partial \pi_n}{\partial x_n} + \text{Ro}^{-1} (v_n - 1) - b_n \sin \alpha + \text{Re}^{-1} \left( \frac{\partial^2 u_n}{\partial x_n^2} + \frac{\partial^2 u_n}{\partial y_n^2} + \frac{\partial^2 u_n}{\partial z_n^2} \right),
\]

\[
\frac{\partial v_n}{\partial t_n} + u_n \frac{\partial v_n}{\partial x_n} + v_n \frac{\partial v_n}{\partial y_n} + w_n \frac{\partial v_n}{\partial z_n} = -\frac{\partial \pi_n}{\partial y_n} - \text{Ro}^{-1} u_n + \text{Re}^{-1} \left( \frac{\partial^2 v_n}{\partial x_n^2} + \frac{\partial^2 v_n}{\partial y_n^2} + \frac{\partial^2 v_n}{\partial z_n^2} \right),
\]

\[
\frac{\partial w_n}{\partial t_n} + u_n \frac{\partial w_n}{\partial x_n} + v_n \frac{\partial w_n}{\partial y_n} + w_n \frac{\partial w_n}{\partial z_n} = -\frac{\partial \pi_n}{\partial z_n} + b_n \cos \alpha + \text{Re}^{-1} \left( \frac{\partial^2 w_n}{\partial x_n^2} + \frac{\partial^2 w_n}{\partial y_n^2} + \frac{\partial^2 w_n}{\partial z_n^2} \right),
\]

\[
\frac{\partial b_n}{\partial t_n} + u_n \frac{\partial b_n}{\partial x_n} + v_n \frac{\partial b_n}{\partial y_n} + w_n \frac{\partial b_n}{\partial z_n} = \text{BuRo}^{-2} (u_n \sin \alpha - w_n \cos \alpha) + \text{Re}^{-1} \left( \frac{\partial^2 b_n}{\partial x_n^2} + \frac{\partial^2 b_n}{\partial y_n^2} + \frac{\partial^2 b_n}{\partial z_n^2} \right),
\]

\[
\frac{\partial u_n}{\partial x_n} + \frac{\partial v_n}{\partial y_n} + \frac{\partial w_n}{\partial z_n} = 0.
\]
Dimensionless parameters of the scaled problem

**Rossby number** \( \text{Ro} = VH^{-1}f^{-1} \)  
**Reynolds number** \( \text{Re} = VH\nu^{-1} \)

**Burger number** \( \text{Bu} = N^2f^{-2} \)  
**\( b_s \)-based Ri number** \( \text{Ri} = -b_sHV^{-2} \)

**\( N \)-based Richardson number** \( \text{BuRo}^{-2} = N^2H^2V^{-2} \)

**Atmosphere (ATM):**

\( V = 10 \text{ m s}^{-1}, \quad H = 10^3 \text{ m}, \quad f = 10^{-4} \text{ s}^{-1}, \quad N = 10^{-2} \text{ s}^{-1}, \quad \nu = 10^{-5} \text{ m}^2\text{s}^{-1}, \quad |b_s| = 0.1 \text{ m s}^{-2}; \)

\( \text{Ro} = 10^2, \quad \text{Re} = 10^9, \quad \text{Bu} = 10^4, \quad |\text{Ri}| = 1 \)

**Length scale range:** \( \eta \sim (\nu^3V^{-3}H)^{1/4} \sim 10^{-4} \text{ m} \quad \leftrightarrow \quad \sim H = 10^3 \text{ m} \)

**Downscaled laboratory analog of atmospheric flow (LEX):**

\( V = 0.1 \text{ m s}^{-1}, \quad H = 0.1 \text{ m}, \quad f = 10^{-2} \text{ s}^{-1}, \quad N = 1 \text{ s}^{-1}, \quad \nu = 10^{-6} \text{ m}^2\text{s}^{-1}, \quad |b_s| = 0.1 \text{ m s}^{-2}; \)

\( \text{Ro} = 10^2, \quad \text{Re} = 10^4, \quad \text{Bu} = 10^4, \quad |\text{Ri}| = 1 \)

**Length scale range:** \( \eta \sim (\nu^3V^{-3}H)^{1/4} \sim 10^{-4} \text{ m} \quad \leftrightarrow \quad \sim H = 0.1 \text{ m} \)
Viscous atmosphere (VAT):

\[ V = 10 \text{ m s}^{-1}, \quad H = 10^3 \text{ m}, \quad f = 10^{-4} \text{s}^{-1}, \quad N = 10^{-2} \text{s}^{-1}, \quad \nu = 1 \text{ m}^2\text{s}^{-1}, \quad |b_s| = 0.1 \text{ m s}^{-2}; \]
\[ \text{Ro} = 10^2, \quad \text{Re} = 10^4, \quad \text{Bu} = 10^4, \quad |\text{Ri}| = 1 \]

Length scale range: \[ \eta \sim (\nu^3 V^{-3} H)^{1/4} \sim 2 \text{ m} \quad \leftrightarrow \quad \sim H = 10^3 \text{ m} \]

Interpretations:

Integral flow scales in VAT are the same as in ATM, but the working fluid is way more viscous than the air.

For given \( \alpha \), the equality of flow numbers (Ro, Re, Bu, and Ri) for the small-scale LEX flow and for the large-scale VAT flow signifies the similarity of these flows in terms of equality of their dimensionless solutions.

VAT simulation may be conceptually interpreted as large-eddy simulation (LES) with constant subgrid eddy diffusivities for momentum and heat/buoyancy.