# A baroclinic nocturnal low-level jet over the Great Plains

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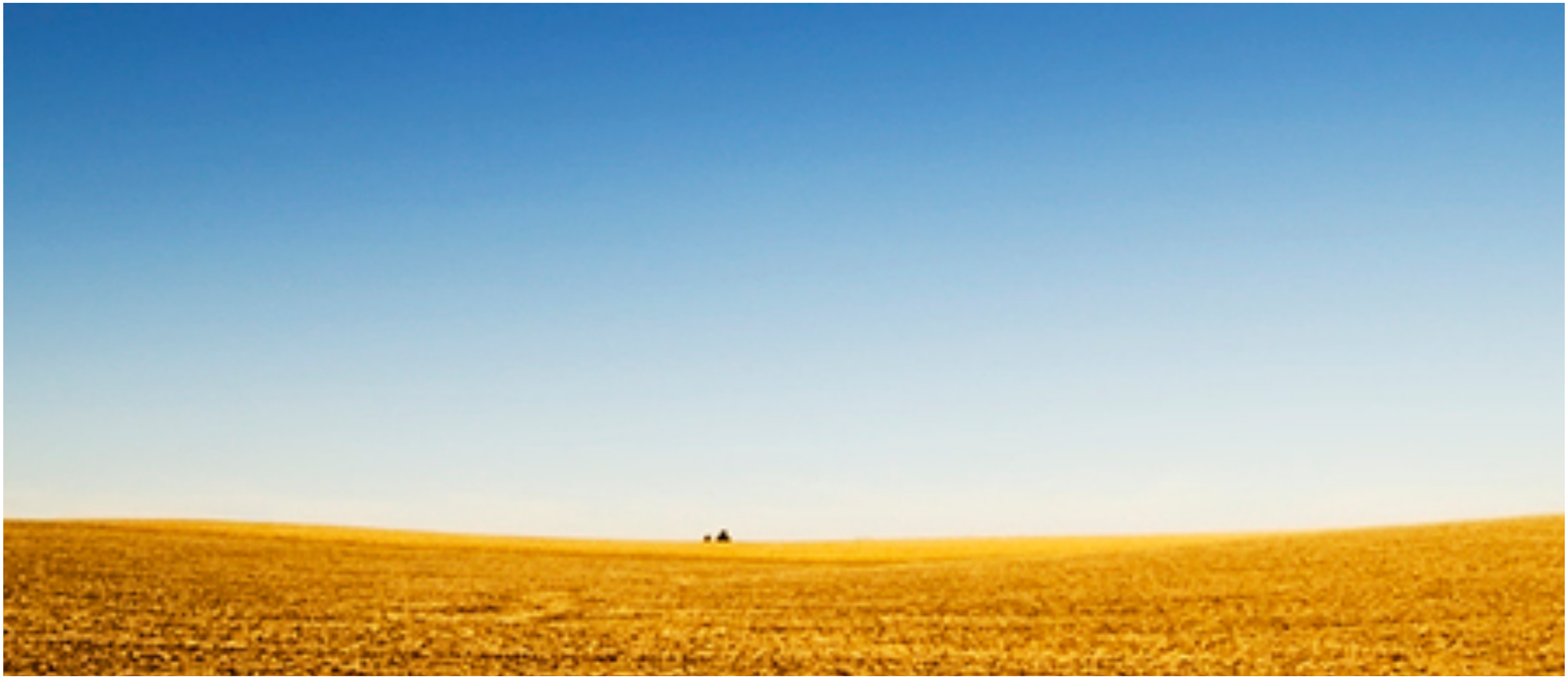
**PECAN Science Workshop, 19 – 21 Sept. 2016, Norman, OK**



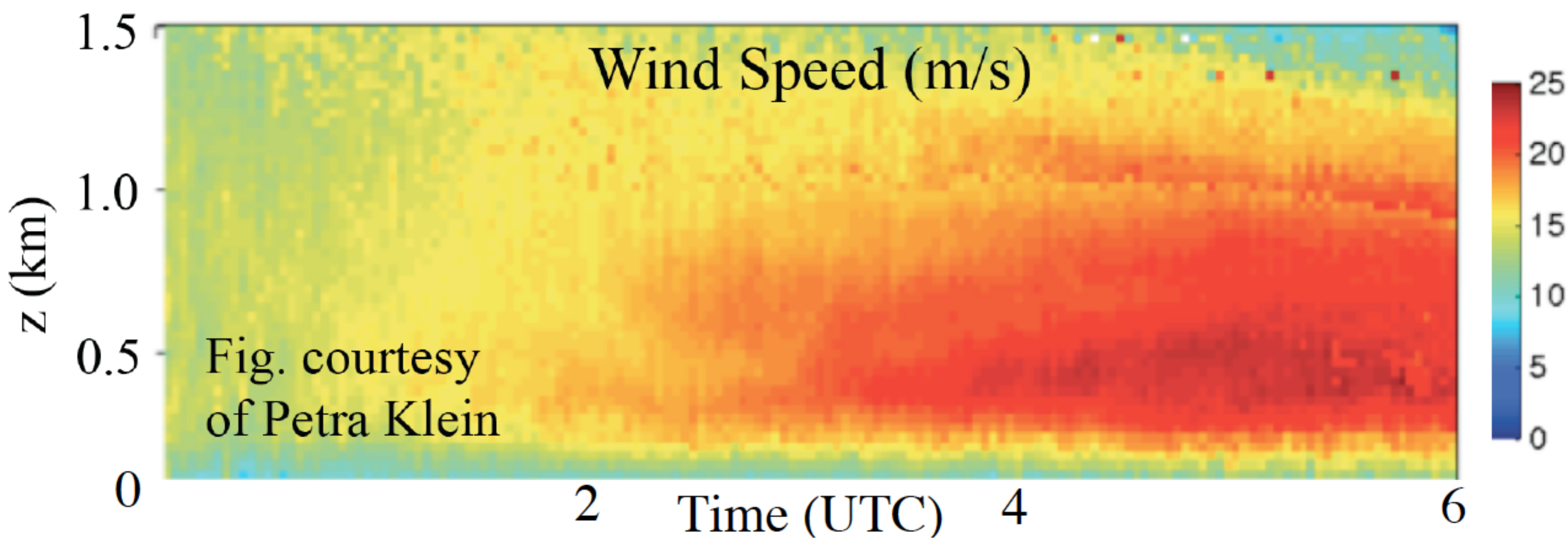
# Characteristics of the Great Plains low-level jet

Setup: **Dry, clear afternoon, southerly geostrophic wind.**

Onset: **Begins near sunset**. **Peaks a few hours after midnight**.

Dissipation: **Mixes out a few hours after dawn**.

Height of peak wind, $z_{max}$, is often **< 500 m**. Can ↑ or ↓ with time.



# Geographical preference (Bonner 1968)

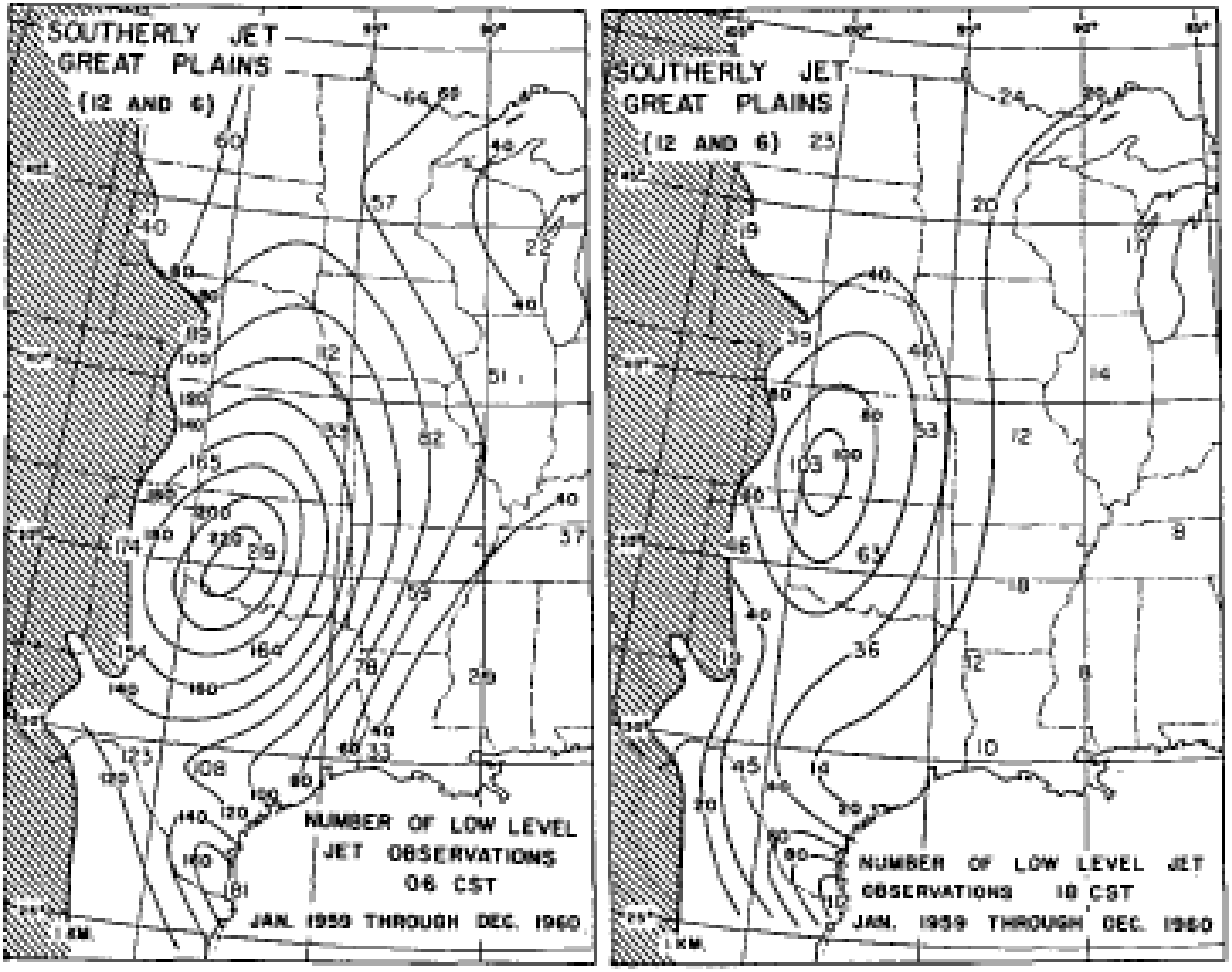
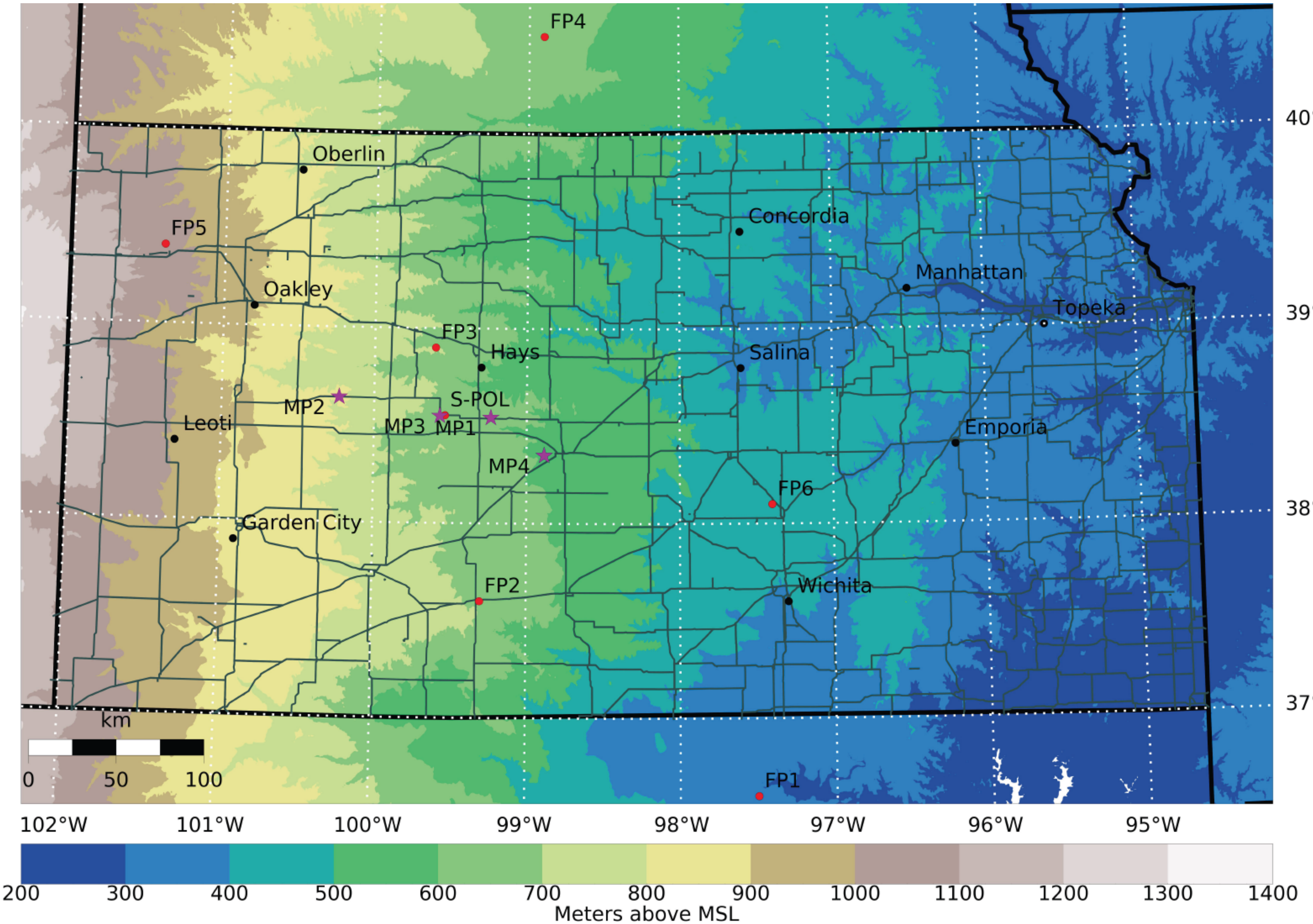



FIGURE 10.—Numbers of Criterion 1 "southerly jet" observations at 06 CST (left) and 18 CST (right). Two years of data.

# Kansas topography



# Theories for the Great Plains low-level jet

## Blackadar (1957) inertial oscillation theory:

**Sudden drop in frictional stress** in CBL around sunset destroys Ekman balance and accelerates air parcels. Coriolis deflection of ageostrophic wind yields an inertial oscillation. Winds peak when the ageostrophic wind aligns with geostrophic wind.

Limitations: Cannot explain geographical preference for the jets. Cannot explain how jet winds exceed $|\mathbf{v}_G|$ by more than 100%.

## Holton (1967) diurnally heated/cooled slope theory:

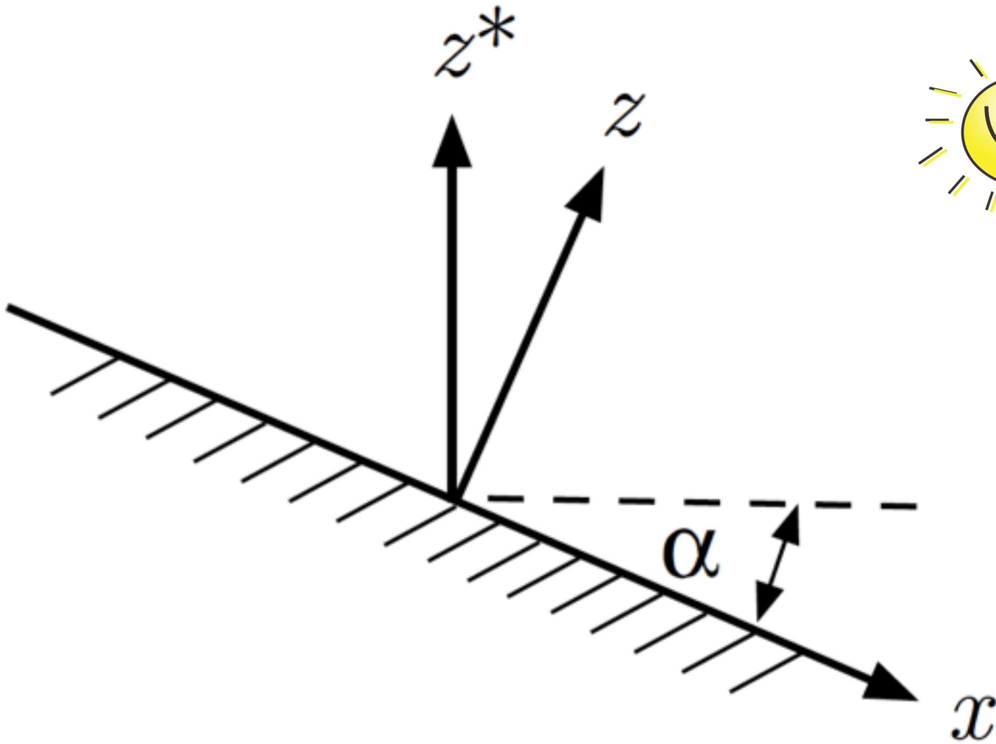
**Diurnal heating cycle on the sloping Great Plains** generates vorticity baroclinically that leads to a diurnal wind oscillation.

Limitations: Does not well-reproduce observed phase of jet oscillations. Does not yield very jet-like flows.

# A unified theory for the Great Plains low-level jet
## [Shapiro, Fedorovich, & Rahimi, JAS, Aug. 2016]

Let eddy viscosity $K(t)$ **decrease suddenly at sunset** (Blackadar)

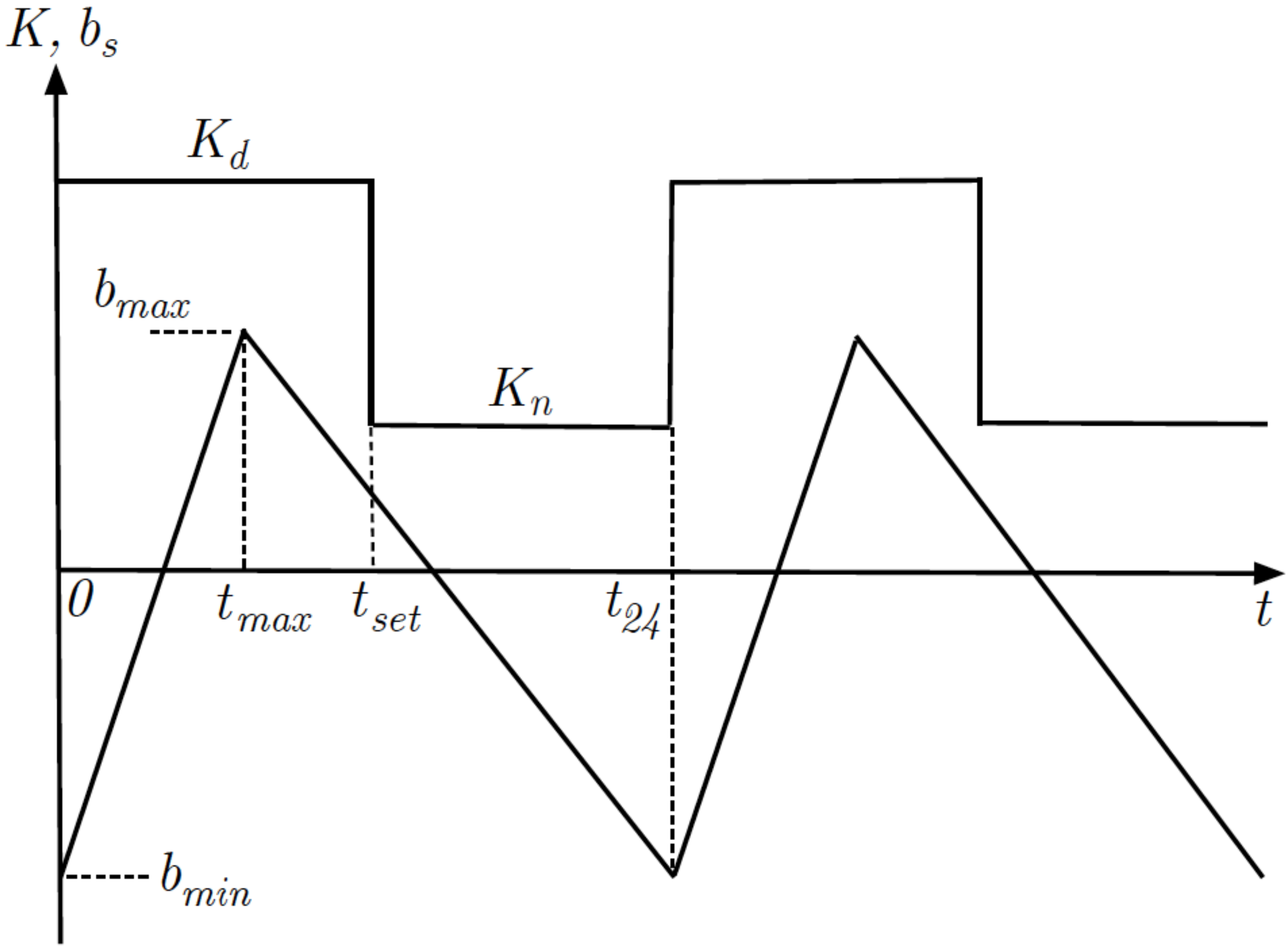
Impose a **slope** with **diurnally varying buoyancy** $b_s(t)$ (Holton)




Include a **thermal energy equation** with provision for **diffusion** and ambient **stratification** [constant $N \equiv \sqrt{(g/\theta_0)d\theta_e/dz^*}$] (Holton)

Consider a **southerly** geostrophic wind $v_G$ (Holton).

# Diurnal oscillations in *K* and $b_s$



# Governing equations

Consider 1D Boussinesq equations of motion and thermal energy:

$$\frac{\partial u}{\partial t} = f v_a - b \sin\alpha + K(t)\frac{\partial^2 u}{\partial z^2}, \tag{1}$$

$$\frac{\partial v_a}{\partial t} = -fu + K(t)\frac{\partial^2 v_a}{\partial z^2}, \tag{2}$$

$$0 = -\frac{\partial \Pi}{\partial z} - b\cos\alpha, \tag{3}$$

$$\frac{\partial b}{\partial t} = uN^2 \sin\alpha - \delta b + K(t)\frac{\partial^2 b}{\partial z^2}. \tag{4}$$

These are similar to Holton's equations, but our *K* is time-dependent and our radiative damping term is simpler: $-\delta b$ ($\delta$ is inverse damping time scale), as in Egger (1985) and Mo (2013).

Slope conditions: no-slip ($u = v = 0$), and specified $b_s(t)$ at *z* = 0.

# A special linear transformation

Following Gutman & Malbakhov (1964), we reduce the governing equations to simpler forms. Taking $\sin\alpha\times(4)+k\times(1)+l\times(2)$ yields:

$$\frac{\partial Q}{\partial t}=kfv_a-b\sin\alpha(k+\delta)+u(N^2\sin^2\alpha-lf)+K\frac{\partial^2 Q}{\partial z^2}, \quad (5)$$

where $k$ and $l$ are constants and $Q$ is a new dependent variable,

$$Q\equiv b\sin\alpha+k\,u+l\,v_a. \quad (6)$$

Find $k$ and $l$ so the red terms in (5) sum to $\mu Q$ ($\mu$ is another constant). Get two $k$ (we only need two) from the cubic equation

$$k^3+2\delta k^2+(f^2+N^2\sin^2\alpha+\delta^2)k+\delta N^2\sin^2\alpha=0, \quad (7)$$

then get two $\mu=-k-\delta$, then get two $l=kf/\mu$, then get two $Q$.

# Transformed governing equations

In terms of the new dependent variables $Q_j$, (1)–(4) reduce to:

$$\frac{\partial Q_j}{\partial t} = \mu_j Q_j + K(t)\frac{\partial^2 Q_j}{\partial z^2}, \qquad j = 1, 2. \tag{8}$$

Plan: get $Q_j$ from (8) then invert (6) to get $u$, $v_a$, $b$ in terms of $Q_j$ as

$$u = -\frac{\mathrm{Im}(l_2)}{\Delta}Q_1 + \frac{\mathrm{Im}(l_2)}{\Delta}\mathrm{Re}(Q_2) + \frac{l_1 - \mathrm{Re}(l_2)}{\Delta}\mathrm{Im}(Q_2), \tag{9}$$

$$v_a = \frac{\mathrm{Im}(k_2)}{\Delta}Q_1 - \frac{\mathrm{Im}(k_2)}{\Delta}\mathrm{Re}(Q_2) - \frac{k_1 - \mathrm{Re}(k_2)}{\Delta}\mathrm{Im}(Q_2), \tag{10}$$

$$b = \text{awful mess}, \tag{11}$$

where $\Delta \equiv \mathrm{Im}(k_2)[l_1 - \mathrm{Re}(l_2)] - \mathrm{Im}(l_2)[k_1 - \mathrm{Re}(k_2)]$.

# Periodic solutions for $Q_j$

Periodic solutions of (8) that vanish as $z \to \infty$ are obtained as

$$Q_j = e^{\mu_j [t - \kappa(t)]} \sum_{m=-\infty}^{\infty} D_{j,m} F_m(t) e^{\pm i z \sqrt{\lambda_{j,m}}}, \qquad j = 1, 2, \qquad (12)$$

where $t_{24} \equiv 24\,\mathrm{h}$, $D_{j,m}$ are unknown coefficients, and

$$F_m(t) \equiv e^{2m\pi i \kappa(t)/t_{24}}, \qquad \kappa(t) \equiv \int_0^t \frac{K(\tau)}{\overline{K}} d\tau,$$

$$\overline{K} \equiv \int_0^{t_{24}} \frac{K(\tau)}{t_{24}} d\tau, \qquad \lambda_{j,m} \equiv \frac{\mu_j - 2m\pi i / t_{24}}{\overline{K}}.$$

The $D_{j,m}$ are determined so that the slope conditions are satisfied.

Takes ~5 min of CPU time to evaluate the solution with $\Delta z = 20\,\mathrm{m}$, $\Delta t = 10\,\mathrm{min}$, and 40,000 terms in the series.

# Test case: PECAN IOP 7 (LLJ), 9-10 June 2015

From "report.chief_scientist.201506092330.summary":

**Report on IOP 7**
**LLJ mission, 9 - 10 June 2015**
**Alan Shapiro**

All times valid on 10 June unless noted as 9 June.

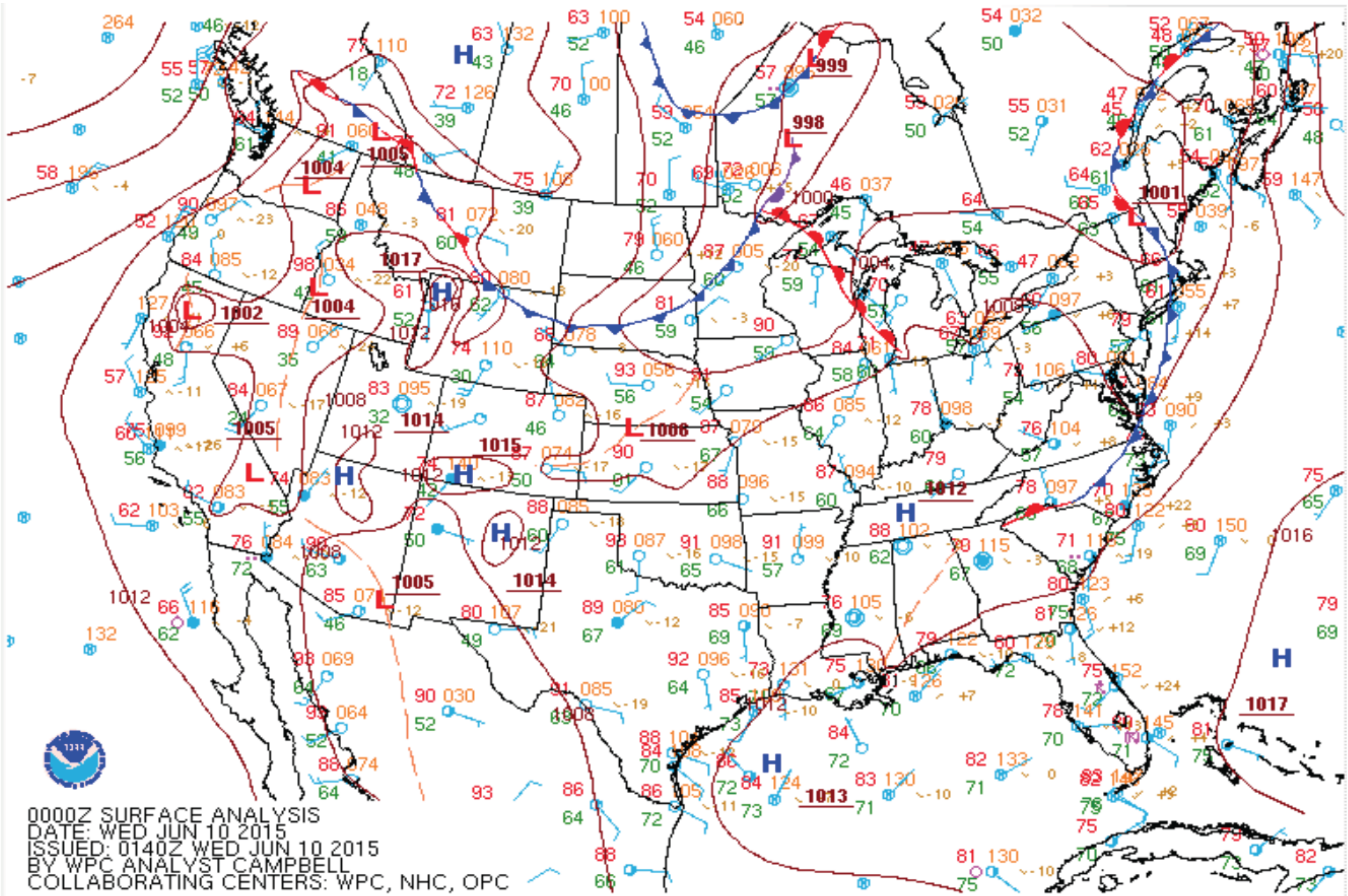
### Forecast guidance

Forecast was for a hot 9 June afternoon followed by the development of a strong but shallow nocturnal LLJ. The predicted LLJ develops late (6 UTC) and covers much of KS, but with notable strengthening west-to-east across KS. Peak winds of 40 knots in NE KS by 0900 UTC. Winds above the jet are weak; even 500 mb flow is weak. No MCSs, bores, or CI in the PECAN domain. The guidance pointed to the development of a classical boundary-layer-forced jet arising from terrain-associated baroclinicity.

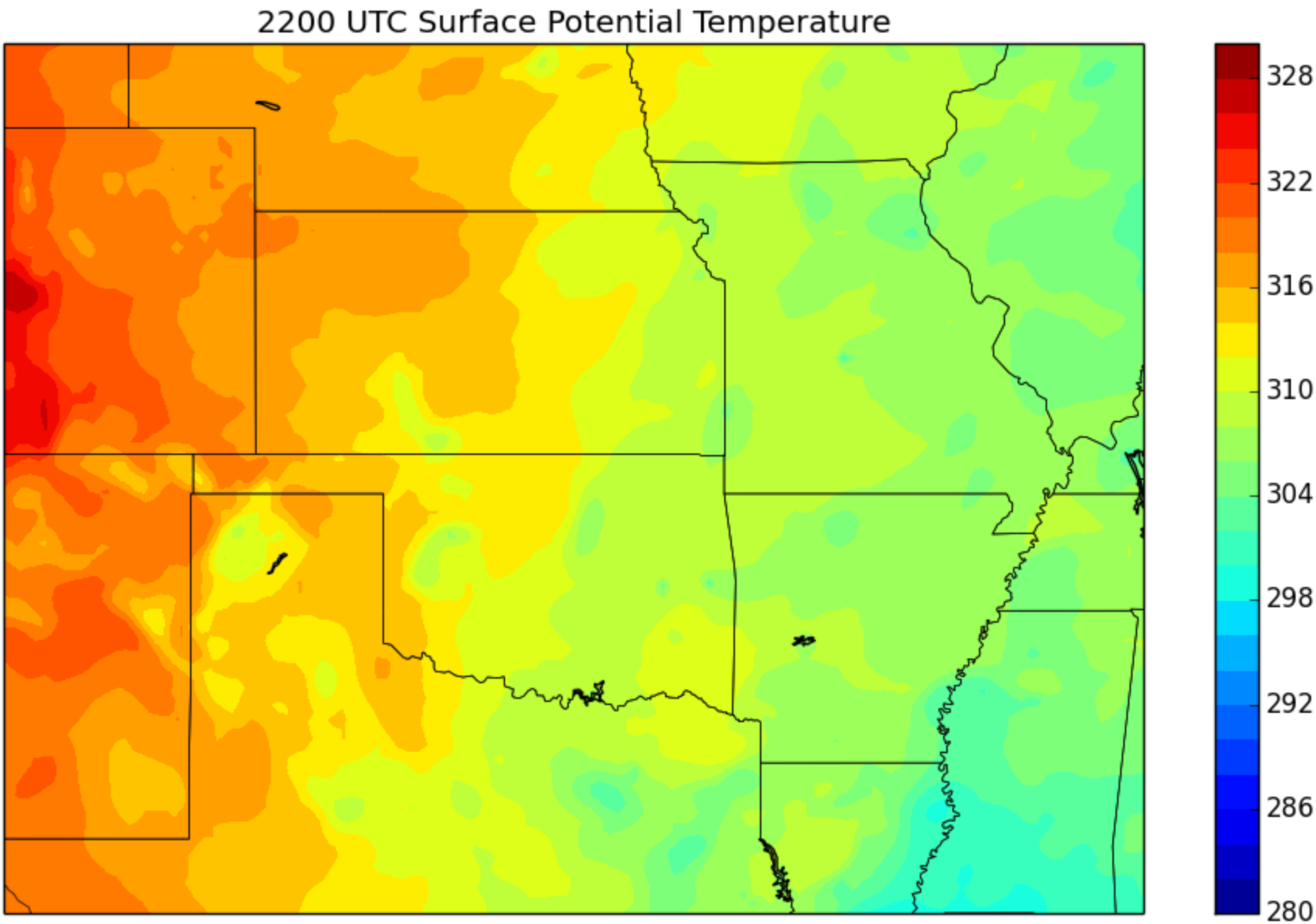
### Planned mission

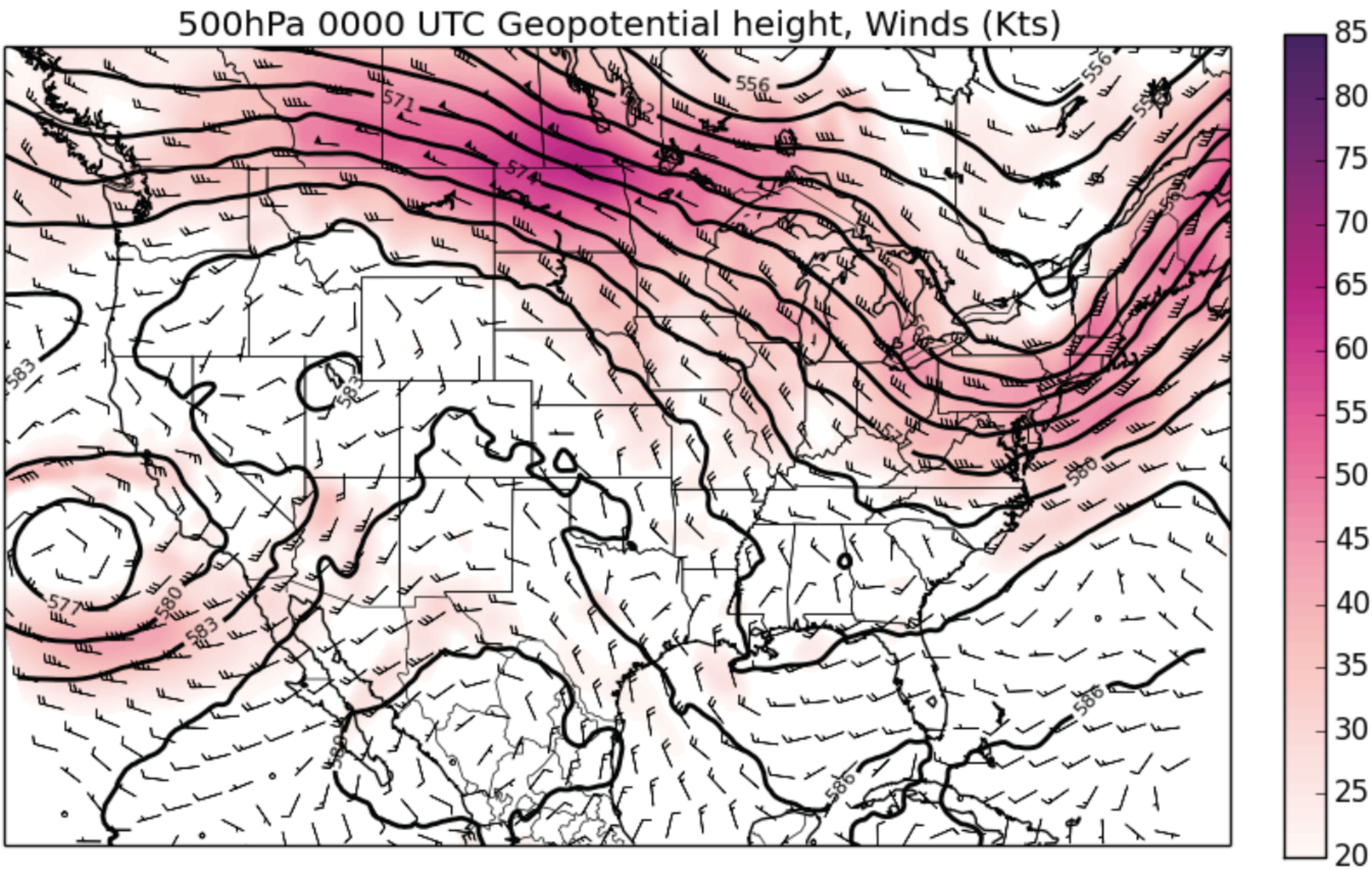
Based on LLJ Scenario 1 (deployments largely as in IOP 2, though with different DOW deployments); domain centered on S-Pol, near McCracken, KS.

# 0000Z Surface Analysis, 10 June 2015



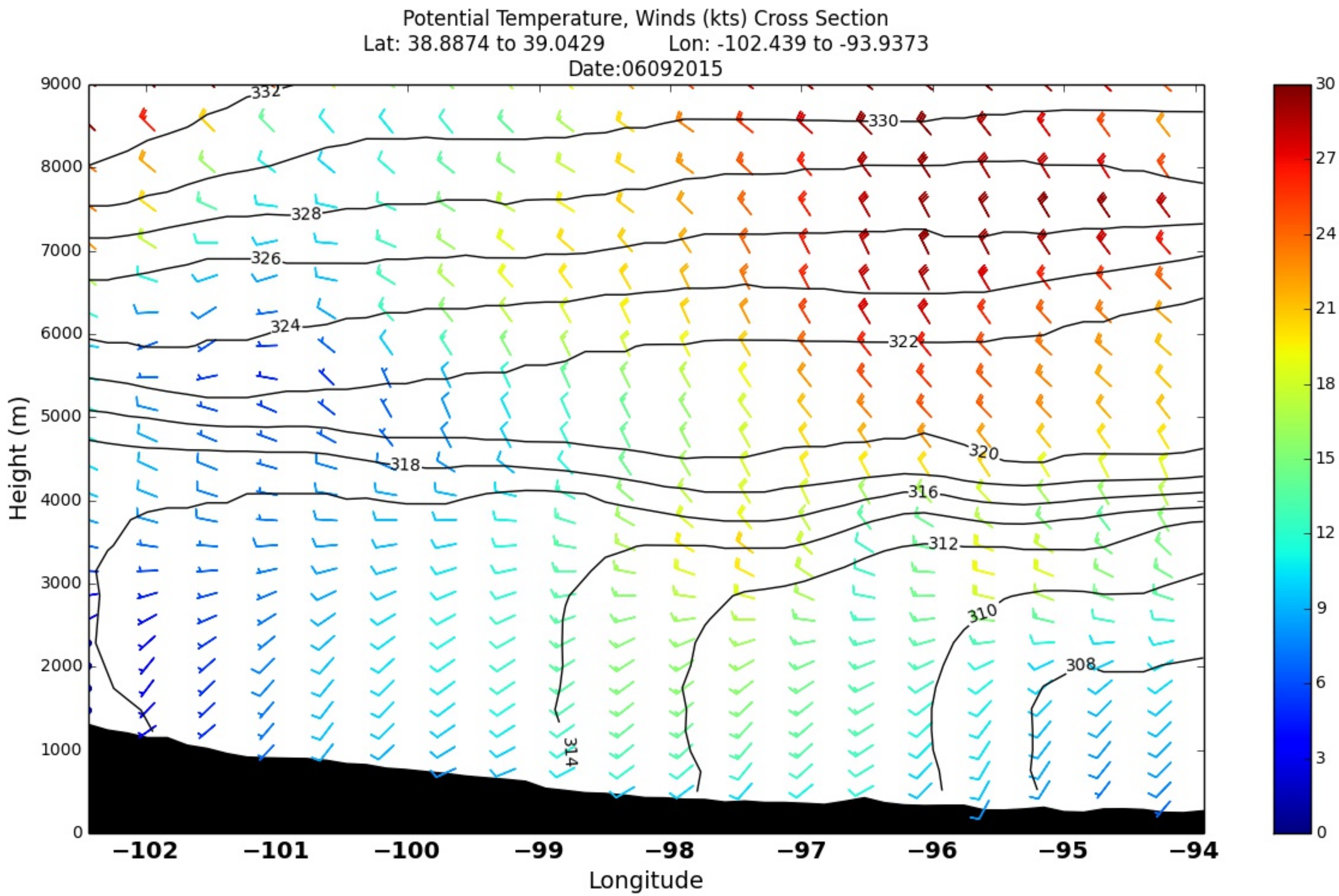
# RAP surface θ analysis, 22 UTC 9 June






At 500 hPa there are weak northwest winds over the PECAN domain. The stronger 500 hPa flow is well north of the domain.

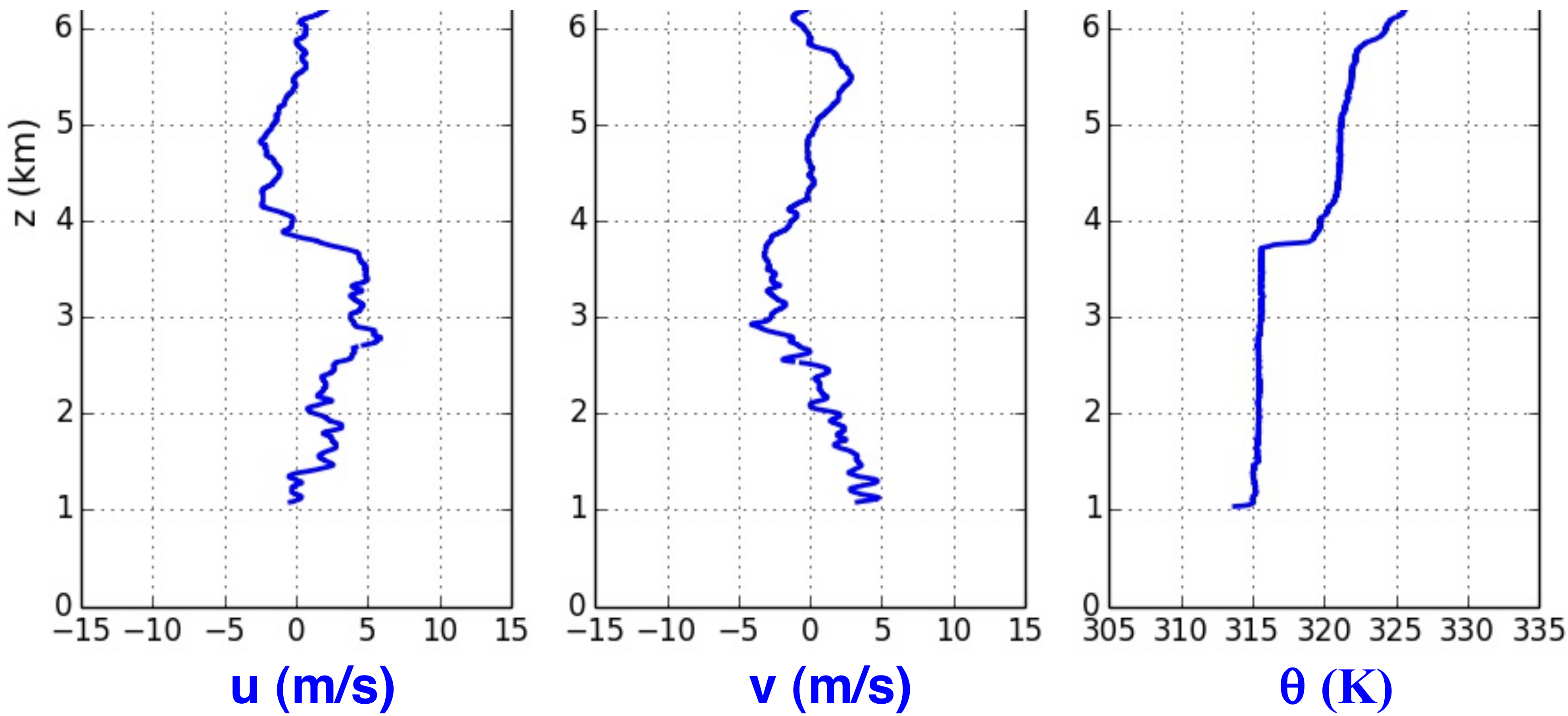
# RAP θ and wind analysis, 39ºN, 22 UTC 9 June

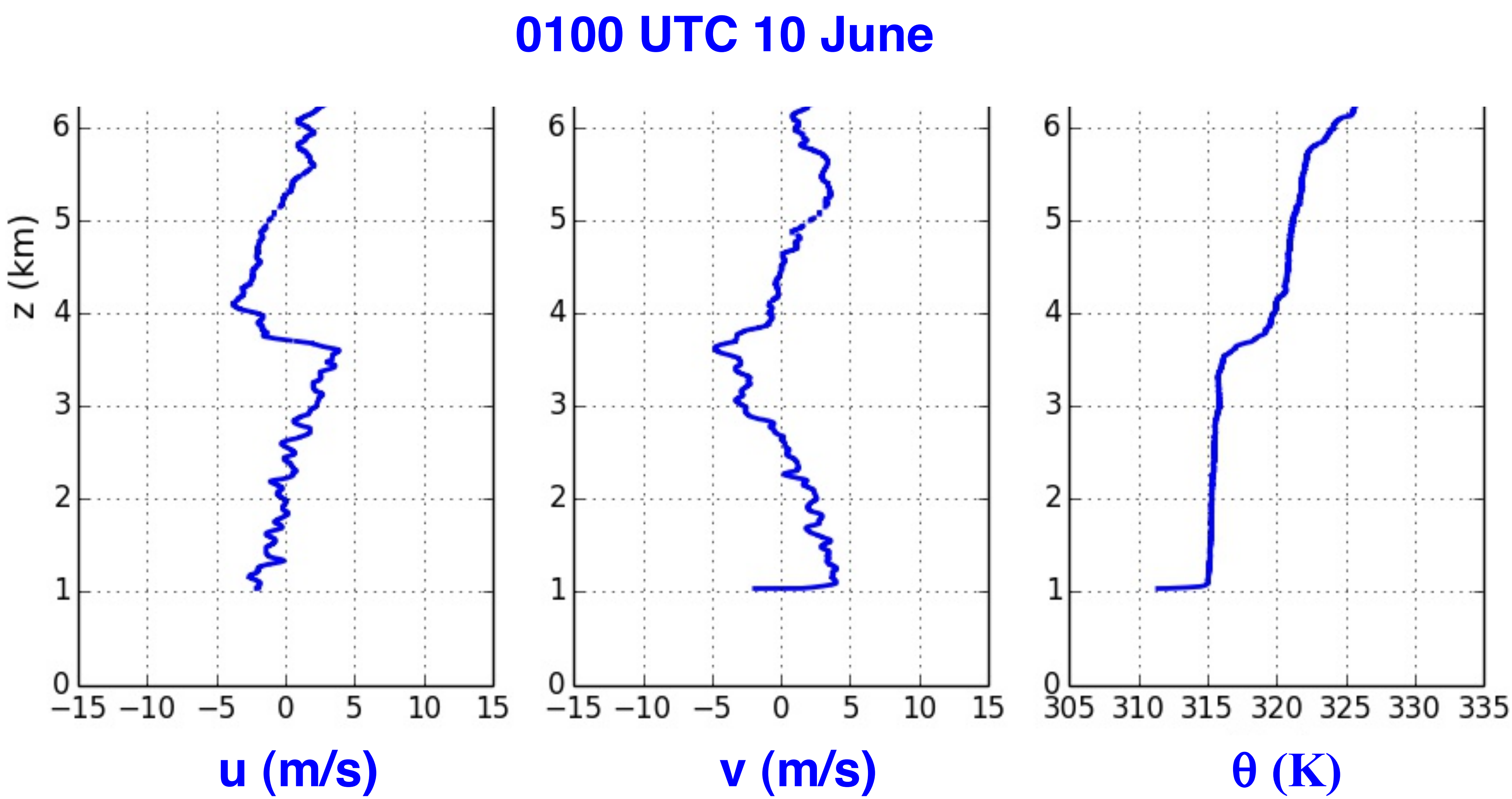


# The low-level jet at Brewster, KS (FP5)

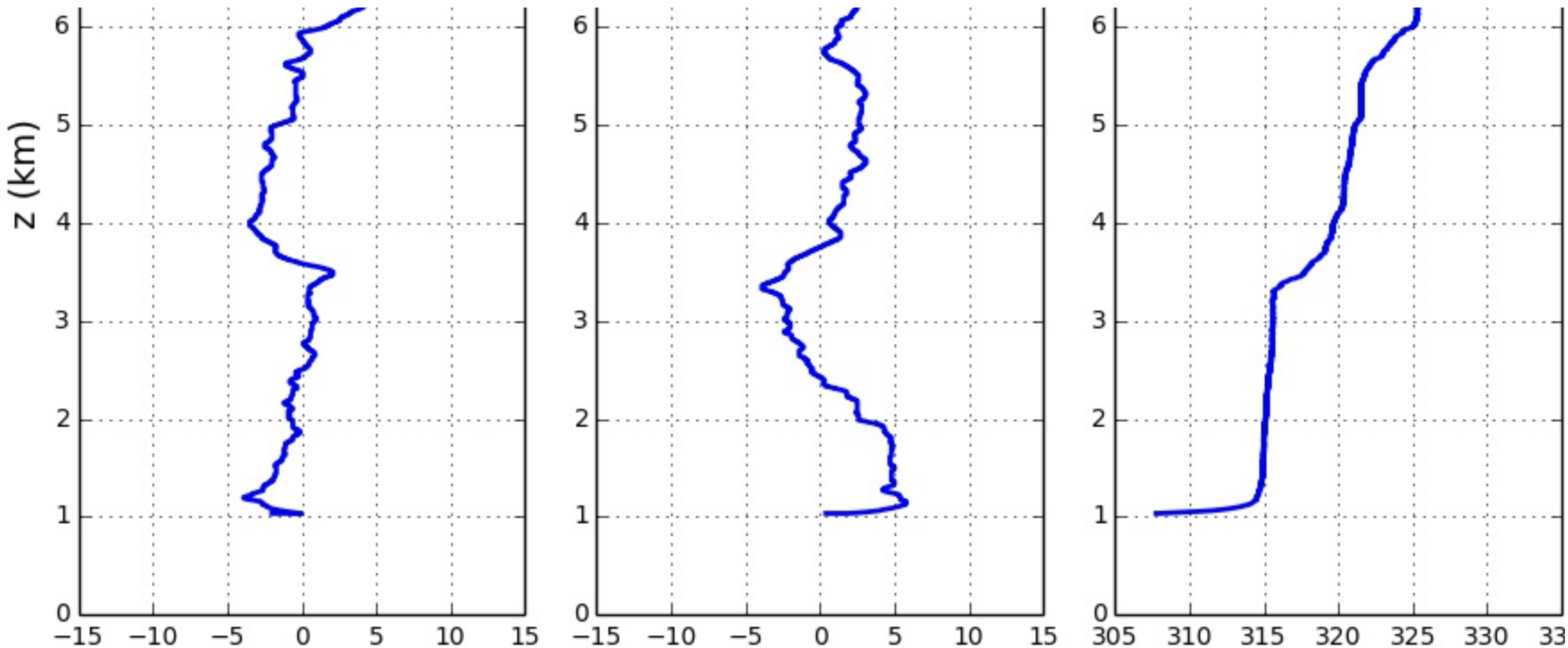
FP5 soundings @ 2330 9 June;  0100, 0230, 0530, 0830 10 June
(Sounding times are in UTC)

**2330 UTC 9 June**



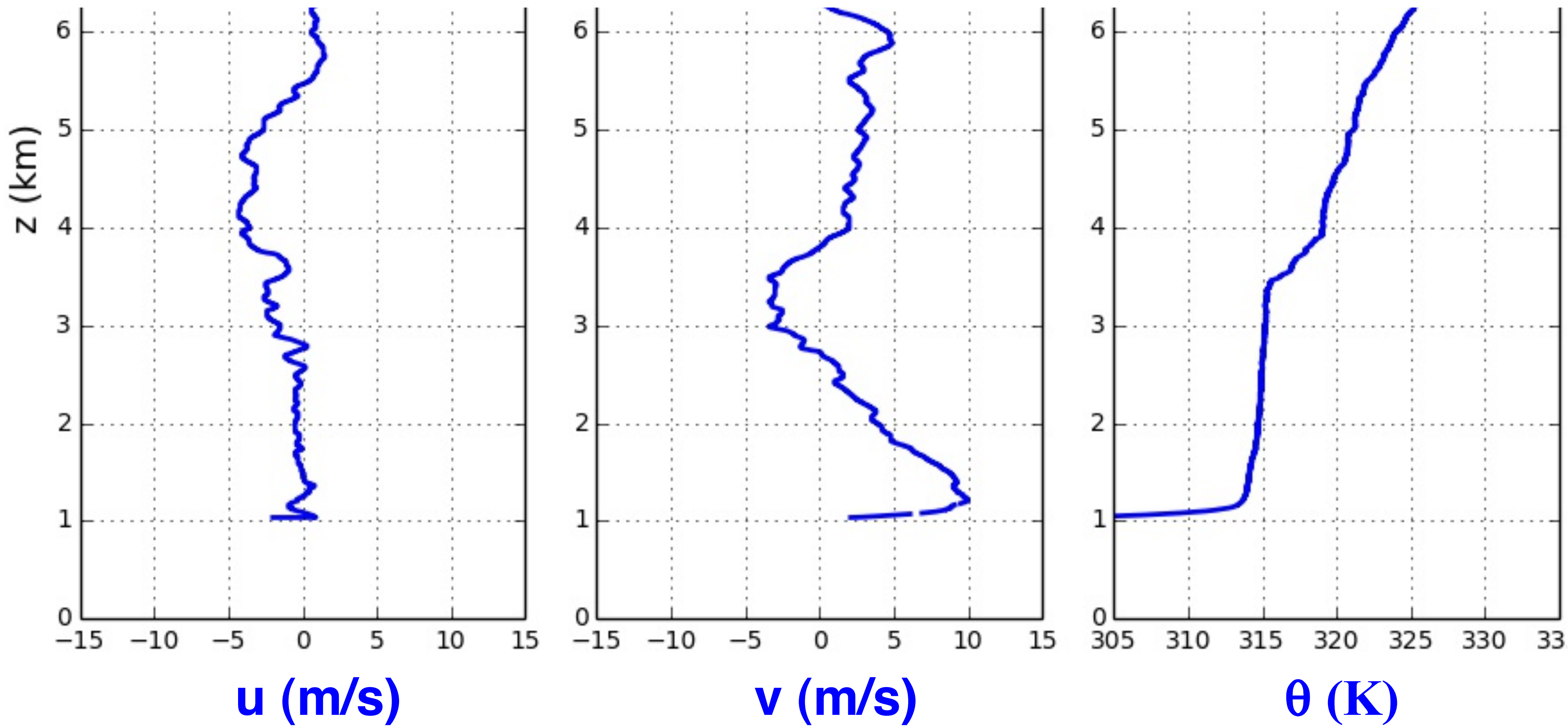

0100 UTC 10 June
z (km)
u (m/s)
v (m/s)
θ (K)

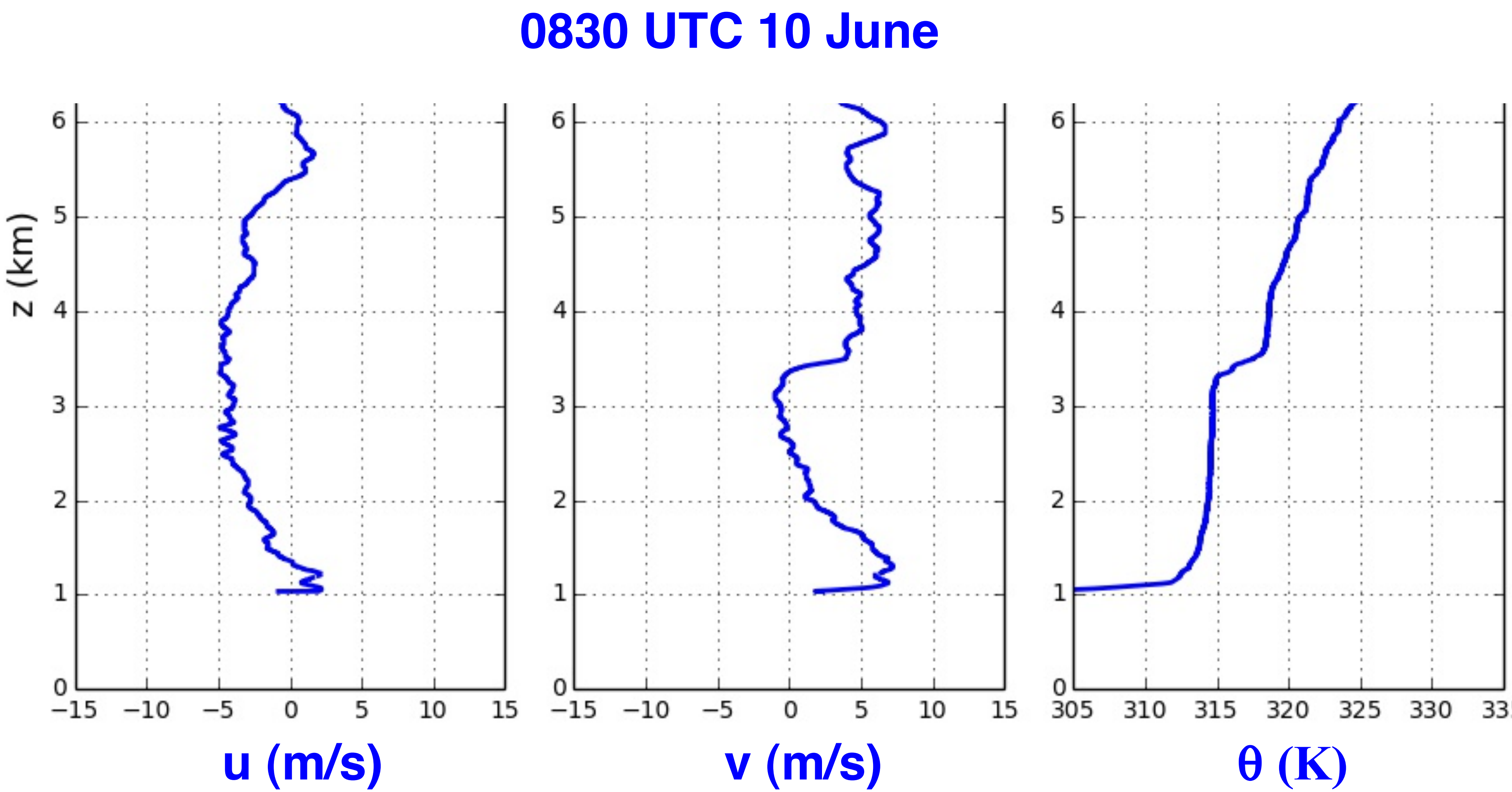
# 0230 UTC 10 June




u (m/s)    v (m/s)    θ (K)

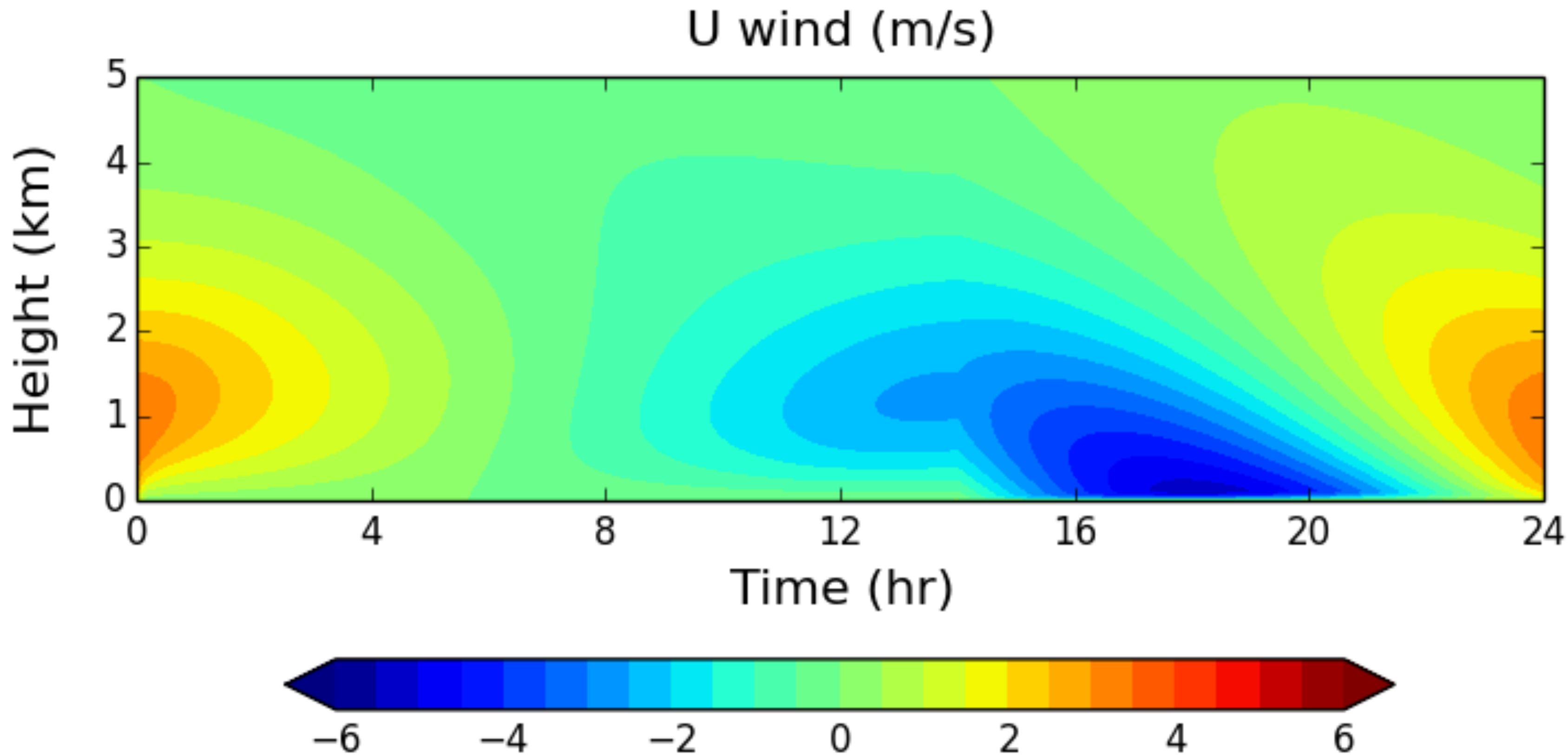
# 0530 UTC 10 June




0830 UTC 10 June
z (km)
0
1
2
3
4
5
6
−15
−10
−5
0
5
10
15
305
310
315
320
325
330
335
u (m/s)
v (m/s)
θ (K)

# Parameters input to analytical solution

| | |
|---|---|
| $f$ | $9.2\times10^{-5}\,\mathrm{s}^{-1}$ (lat$=39^\circ$) |
| $v_G$ | $0\,\mathrm{m\,s}^{-1}$ |
| $\alpha$ | $0.15^\circ$ |
| $N$ | $0.01\,\mathrm{s}^{-1}$ |
| $b_{\max}$ | $0.33\,\mathrm{m\,s}^{-2}$ |
| $b_{\min}$ | $-0.20\,\mathrm{m\,s}^{-2}$ |
| $t_{\max}$ | 11 h after sunrise |
| $t_{\mathrm{set}}$ | 14 h after sunrise (a fudged 15 h) |
| $K_d$ | $100\,\mathrm{m}^2\,\mathrm{s}^{-1}$ |
| $K_n$ | $0.1\,\mathrm{m}^2\,\mathrm{s}^{-1}$ |
| $\delta$ | $(5\,\mathrm{d})^{-1}$ |




Time on plot is **hrs after sunrise. Sunrise is at ~ 5 CST (11 UTC)**

1st sounding @ 2330 UTC = 12:30 h after sunrise

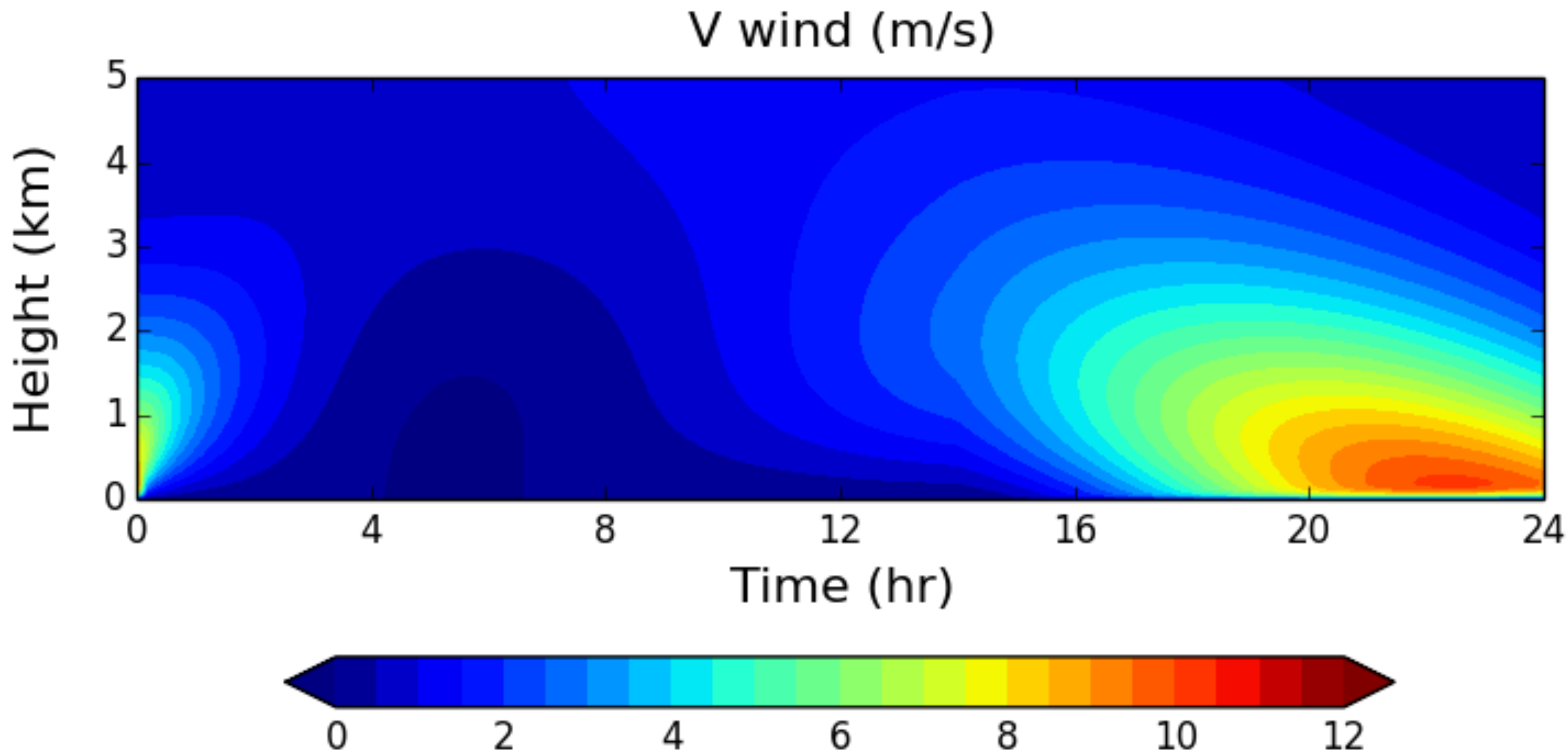
2nd sounding @ 0100 UTC = 14:00 h after sunrise

3rd sounding @ 0230 UTC = 15:30 h after sunrise

4th sounding @ 0530 UTC = 18:30 h after sunrise

5th sounding @ 0830 UTC = 21:30 h after sunrise

**--> Low level upslope flow (u < 0) followed by downslope flow (u > 0) verify but the timing is off (delayed)**

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5th sounding @ 0830 UTC = 21:30 h after sunrise

**--> Peak v of 10 m/s @ 200 m AGL verifies but occurs too late.**