A baroclinic nocturnal low-level jet over the Great Plains

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Characteristics of the Great Plains low-level jet

Setup: Dry, clear afternoon, southerly geostrophic wind.

Onset: Begins near sunset. Peaks a few hours after midnight.

Dissipation: Mixes out a few hours after dawn.

Height of peak wind, $z_{\text{max}}$, is often $<500 \text{ m}$. Can ↑ or ↓ with time.
Geographical preference (Bonner 1968)

Figure 10.—Numbers of Criterion 1 “southerly jet” observations at 06 CST (left) and 18 CST (right). Two years of data.
Kansas topography
Theories for the Great Plains low-level jet

Blackadar (1957) inertial oscillation theory:

Sudden drop in frictional stress in CBL around sunset destroys Ekman balance and accelerates air parcels. Coriolis deflection of ageostrophic wind yields an inertial oscillation. Winds peak when the ageostrophic wind aligns with geostrophic wind.

Limitations: Cannot explain geographical preference for the jets. Cannot explain how jet winds exceed $|v_G|$ by more than 100%.

Holton (1967) diurnally heated/cooled slope theory:

Diurnal heating cycle on the sloping Great Plains generates vorticity baroclinically that leads to a diurnal wind oscillation.

Limitations: Does not well-reproduce observed phase of jet oscillations. Does not yield very jet-like flows.
A unified theory for the Great Plains low-level jet
[Shapiro, Fedorovich, & Rahimi, JAS, Aug. 2016]

Let eddy viscosity $K(t)$ decrease suddenly at sunset (Blackadar)

Impose a slope with diurnally varying buoyancy $b_s(t)$ (Holton)

Include a thermal energy equation with provision for diffusion and ambient stratification [constant $N \equiv \sqrt{(g/\theta_0)d\theta_e/dz^*}$] (Holton)

Consider a southerly geostrophic wind $v_G$ (Holton).
Diurnal oscillations in $K$ and $b_s$
Governing equations

Consider 1D Boussinesq equations of motion and thermal energy:

\[
\frac{\partial u}{\partial t} = f v_a - b \sin \alpha + K(t) \frac{\partial^2 u}{\partial z^2},
\]  
(1)

\[
\frac{\partial v_a}{\partial t} = -f u + K(t) \frac{\partial^2 v_a}{\partial z^2},
\]  
(2)

\[\Pi_t = -\frac{\partial \Pi}{\partial z} - b \cos \alpha,\]  
(3)

\[
\frac{\partial b}{\partial t} = u N^2 \sin \alpha - \delta b + K(t) \frac{\partial^2 b}{\partial z^2}.
\]  
(4)

These are similar to Holton's equations, but our \(K\) is time-dependent and our radiative damping term is simpler: \(-\delta b\) (\(\delta\) is inverse damping time scale), as in Egger (1985) and Mo (2013).

Slope conditions: no-slip \((u = v = 0)\), and specified \(b_s(t)\) at \(z = 0\).
A special linear transformation

Following Gutman & Malbakhov (1964), we reduce the governing equations to simpler forms. Taking \( \sin \alpha \times (4) + k \times (1) + l \times (2) \) yields:

\[
\frac{\partial Q}{\partial t} = k f v_a - b \sin \alpha (k + \delta) + u (N^2 \sin^2 \alpha - l f) + K \frac{\partial^2 Q}{\partial z^2},
\]  

(5)

where \( k \) and \( l \) are constants and \( Q \) is a new dependent variable,

\[
Q \equiv b \sin \alpha + k u + l v_a.
\]  

(6)

Find \( k \) and \( l \) so the red terms in (5) sum to \( \mu Q \) (\( \mu \) is another constant). Get two \( k \) (we only need two) from the cubic equation

\[
k^3 + 2 \delta k^2 + (f^2 + N^2 \sin^2 \alpha + \delta^2)k + \delta N^2 \sin^2 \alpha = 0,
\]  

(7)

then get two \( \mu = -k - \delta \), then get two \( l = kf/\mu \), then get two \( Q \).
Transformed governing equations

In terms of the new dependent variables $Q_j$, (1)–(4) reduce to:

$$\frac{\partial Q_j}{\partial t} = \mu_j Q_j + K(t) \frac{\partial^2 Q_j}{\partial z^2}, \quad j = 1, 2. \quad (8)$$

Plan: get $Q_j$ from (8) then invert (6) to get $u$, $v_a$, $b$ in terms of $Q_j$ as

$$u = -\frac{\text{Im}(l_2)}{\Delta} Q_1 + \frac{\text{Im}(l_2)}{\Delta} \text{Re}(Q_2) + \frac{l_1 - \text{Re}(l_2)}{\Delta} \text{Im}(Q_2), \quad (9)$$

$$v_a = \frac{\text{Im}(k_2)}{\Delta} Q_1 - \frac{\text{Im}(k_2)}{\Delta} \text{Re}(Q_2) - \frac{k_1 - \text{Re}(k_2)}{\Delta} \text{Im}(Q_2), \quad (10)$$

$$b = \text{awful mess}, \quad (11)$$

where $\Delta \equiv \text{Im}(k_2)[l_1 - \text{Re}(l_2)] - \text{Im}(l_2)[k_1 - \text{Re}(k_2)]$. 

Periodic solutions for $Q_j$

Periodic solutions of (8) that vanish as $z \to \infty$ are obtained as

$$Q_j = e^{\mu_j [t - \kappa(t)]} \sum_{m=-\infty}^{\infty} D_{j,m} F_m(t) e^{\pm iz \sqrt{\lambda_{j,m}}}, \quad j = 1, 2, \quad (12)$$

where $t_{24} \equiv 24 \text{ h}$, $D_{j,m}$ are unknown coefficients, and

$$F_m(t) \equiv e^{2m\pi i \kappa(t)/t_{24}}, \quad \kappa(t) \equiv \int_0^t \frac{K(\tau)}{K} \, d\tau,$$

$$\overline{K} \equiv \int_0^{t_{24}} \frac{K(\tau)}{t_{24}} \, d\tau, \quad \lambda_{j,m} \equiv \frac{\mu_j - 2m\pi i / t_{24}}{\overline{K}}.$$

The $D_{j,m}$ are determined so that the slope conditions are satisfied.

Takes $\sim 5 \text{ min}$ of CPU time to evaluate the solution with $\Delta z = 20 \text{ m}$, $\Delta t = 10 \text{ min}$, and 40,000 terms in the series.
Test case: PECAN IOP 7 (LLJ), 9-10 June 2015

From "report.chief_scientist.201506092330.summary":

Report on IOP 7
LLJ mission, 9 - 10 June 2015
Alan Shapiro

All times valid on 10 June unless noted as 9 June.

Forecast guidance
Forecast was for a hot 9 June afternoon followed by the development of a strong but shallow nocturnal LLJ. The predicted LLJ develops late (6 UTC) and covers much of KS, but with notable strengthening west-to-east across KS. Peak winds of 40 knots in NE KS by 0900 UTC. Winds above the jet are weak; even 500 mb flow is weak. No MCSs, bores, or CI in the PECAN domain. The guidance pointed to the development of a classical boundary-layer-forced jet arising from terrain-associated baroclinicity.

Planned mission
Based on LLJ Scenario 1 (deployments largely as in IOP 2, though with different DOW deployments); domain centered on S-Pol, near McCracken, KS.
RAP surface $\theta$ analysis, 22 UTC 9 June
At 500 hPa there are weak northwest winds over the PECAN domain. The stronger 500 hPa flow is well north of the domain.
RAP $\theta$ and wind analysis, 39°N, 22 UTC 9 June
The low-level jet at Brewster, KS (FP5)

FP5 soundings @ 2330 9 June; 0100, 0230, 0530, 0830 10 June
(Sounding times are in UTC)

2330 UTC 9 June

\[ u \text{ (m/s)} \]  \[ v \text{ (m/s)} \]  \[ \theta \text{ (K)} \]
0100 UTC 10 June

- $u (\text{m/s})$
- $v (\text{m/s})$
- $\theta (\text{K})$
0230 UTC 10 June

Graphs showing:
- u (m/s)
- v (m/s)
- θ (K)
0530 UTC 10 June

\[ Z \text{ (km)} \]

\[ u \text{ (m/s)} \]

\[ v \text{ (m/s)} \]

\[ \theta \text{ (K)} \]
0830 UTC 10 June

- $u$ (m/s)
- $v$ (m/s)
- $\theta$ (K)
## Parameters input to analytical solution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>$9.2 \times 10^{-5} \text{s}^{-1}$ (lat = $39^\circ$)</td>
</tr>
<tr>
<td>$v_G$</td>
<td>0 m s$^{-1}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$0.15^\circ$</td>
</tr>
<tr>
<td>$N$</td>
<td>0.01 s$^{-1}$</td>
</tr>
<tr>
<td>$b_{\text{max}}$</td>
<td>0.33 m s$^{-2}$</td>
</tr>
<tr>
<td>$b_{\text{min}}$</td>
<td>$-0.20$ m s$^{-2}$</td>
</tr>
<tr>
<td>$t_{\text{max}}$</td>
<td>11 h after sunrise</td>
</tr>
<tr>
<td>$t_{\text{set}}$</td>
<td>14 h after sunrise (a fudged 15 h)</td>
</tr>
<tr>
<td>$K_d$</td>
<td>100 m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$K_n$</td>
<td>0.1 m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$(5 \text{d})^{-1}$</td>
</tr>
</tbody>
</table>
Time on plot is **hrs after sunrise. Sunrise is at ~ 5 CST (11 UTC)**

1st sounding @ 2330 UTC = 12:30 h after sunrise
2nd sounding @ 0100 UTC = 14:00 h after sunrise
3rd sounding @ 0230 UTC = 15:30 h after sunrise
4th sounding @ 0530 UTC = 18:30 h after sunrise
5th sounding @ 0830 UTC = 21:30 h after sunrise

--> Low level upslope flow (u < 0) followed by downslope flow (u > 0) verify but the timing is off (delayed)
1st sounding @ 2330 UTC = 12:30 h after sunrise
2nd sounding @ 0100 UTC = 14:00 h after sunrise
3rd sounding @ 0230 UTC = 15:30 h after sunrise
4th sounding @ 0530 UTC = 18:30 h after sunrise
5th sounding @ 0830 UTC = 21:30 h after sunrise

--> Peak v of 10 m/s @ 200 m AGL verifies but occurs too late.